Collision Broadening of the ϕ Meson in Baryon Rich Hadronic Matter

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Abstract

Phi meson-baryon cross sections, estimated within a one-boson-exchange model, serve as input for a calculation of the collision rates in hot hadronic matter. We find that the width of the ϕ meson is modified through collisions with baryons by 1–10 MeV at 160 MeV temperature depending on the baryon fugacity. Thermalization of the ϕ in high energy heavy ion collisions is discussed.

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I. INTRODUCTION

One of the premier goals of relativistic heavy-ion physics is to identify and describe the quark gluon plasma (QGP). There has been much work directed towards quantifying the properties of QGP [1–4]. Understanding the physics in absence of plasma is necessary preparation. Vector mesons provide a useful probe in measuring the properties of the medium, and in detecting the predicted crossover to the QGP. The ϕ is especially promising because it decays both to kaon pairs ($\sim 83\%$) and, more rarely, to lepton pairs, both of which are readily detectable in high energy nuclear experiments [5]. It is clear, however, that the in-medium properties of the ϕ are most directly detected through its leptonic decays [6]. Reconstructing the ϕ mass through the products of the dominant branching ratio, $\phi \to KK$, does not suffice since detectable kaons emerge at freezeout; kaons from ϕ decays in the medium are unlikely to escape without reacting further, thus destroying any useful information possessed about ϕ. The decay of ϕ to leptons, however, is sufficiently rare that previous experiments have not been able to fully exploit the ϕ as an in-medium probe. The greater luminosities provided by the next generation of colliders should render the ϕ an invaluable tool in quantifying in medium properties.

The ϕ makes a nice probe since it is not masked behind other resonances in mass spectra. Whence, much work has been done predicting how the properties of the ϕ change in the medium. Due to partial restoration of chiral symmetry [7], the mass of the ϕ may be altered. Such studies have been carried out intensively using a variety of methods. The ϕ mass has been studied through the use of effective chiral Lagrangian approaches [8–10] notably through the use of a full SU(3) chiral Lagrangian with $\mathrm{SU}(3)_v$-breaking effects [11], using QCD sum rules [12,13], the Nambu-Jona-Lasinio model [14], and with others. These approaches tend to report a modest drop in the mass of the ϕ with increasing temperature.

The topic of this paper, the width of the ϕ, has also been studied previously. These studies, however, have focused on effects resulting from a ϕ mass dropping scenario due to chiral symmetry restoration, or similarly, through kaon mass dropping effects [11,13,15–19]. This work presents a systematic study of ϕ dynamics by calculating the change in the width of the ϕ through scattering with other particles in the medium. We include a rather complete set of important ϕ-meson scattering reactions which contribute to the collision broadening. In contrast with the earlier works listed, we do not compute a modification in the *decay* width of the ϕ, rather, we calculate a modification in the full width due to vigorous scattering. The bare mass for the ϕ is taken throughout, and dropping mass effects are not considered here. A similar collision broadening study has been carried out for the ρ meson in which mass dropping was considered [20].

II. THE MODEL

We examine conditions typical of heavy-ion collisions at the Alternating Gradient Synchrotron (AGS) up to the Super Proton Synchrotron (SPS). Collisions of these types reach temperatures ranging from 120-160 MeV or higher. Recently observed pion-proton ratios suggest interpretation of a baryon chemical potential in the neighborhood of 400 MeV [21]. We take this as one possible value in our study.

The calculation discussed arises from an effective kinetic theory, based on a fireball model. We approximate distributions of the various particles as Boltzmann.

A. Particle Density

The probability that a ϕ meson will collide with another particle depends upon both the reaction cross section and the population density of the partner. We assume Boltzmann distributions for simplicity. The density is a function of temperature, and for particle i is given by

$$\frac{dn_i}{d^3x} = d_i \int_0^\infty \frac{d^3p_i}{(2\pi)^3} e^{-\beta E}, \tag{2.1}$$

where β is inverse temperature and d_i is the degeneracy. The degeneracy is given by,

$$d_i = (2I_i + 1)(2S_i + 1), \tag{2.2}$$

where I_i is the isospin of the particle, and S_i is the spin. For the ϕ, Eq. (2.1) can be simplified to read

$$\frac{dn_\phi}{d^3x} = \frac{3m_\phi^2 K_2(\frac{m_\phi}{T})T}{2\pi^2}, \tag{2.3}$$

where K_2 is a second order modified Bessel function.

B. The Reaction Cross Section

A fundamental ingredient in the calculation is the reaction cross section. Here we use boson exchange to model the interactions. Starting with the Lagrangians,

$$\mathcal{L}_{NN\pi} = g_{NN\pi}\bar{\psi}\gamma_5\vec{\tau}\psi \cdot \vec{\pi} \tag{2.4}$$

$$\mathcal{L}_{NN\rho} = g_{NN\rho}\bar{\psi}\gamma_\mu\vec{\tau}\psi \cdot \vec{\rho}^\mu + \frac{f_{NN\rho}}{4m_N}\bar{\psi}\sigma_{\mu\nu}\psi\vec{\rho}^{\mu\nu} \tag{2.5}$$

$$\mathcal{L}_{N\Delta\pi} = \frac{f_{N\Delta\pi}}{m_\pi}\bar{\psi}\vec{T}\psi_\mu \cdot \partial^\mu\vec{\pi} \tag{2.6}$$

$$\mathcal{L}_{\phi\rho\pi} = \frac{g_{\phi\rho\pi}}{m_\pi}\epsilon_{\mu\nu\alpha\beta}\partial^\mu\rho^\nu\partial^\alpha\phi^\beta\pi \tag{2.7}$$

$$\mathcal{L}_{KK\phi} = g_{\phi KK}\phi^\mu(\partial_\mu\vec{K} \times \vec{K})_{(3)}, \tag{2.8}$$

we generate a set of tree-level graphs to compute the squared amplitude. The cross section is then computed with

$$\sigma = \frac{1}{F}I(s), \tag{2.9}$$

where

$$F = 2\lambda^{1/2}(s, m_a^2, m_b^2)(2\pi)^2 \tag{2.10}$$

is the flux factor[1] and

$$I_2(s) = \int \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\,\delta^4\left(p_a + p_b - p_1 - p_2\right)|\mathcal{M}|^2 \tag{2.11}$$

contains the integration over phase space.

The model thus far has not taken into account the fact that the particles involved are extended objects. A multiplicative factor must be inserted to suppress high momentum transfers. Each vertex in a t-channel Feynman diagram requires a multiplicative factor attached to the scattering amplitude of the form [24],

$$h = \frac{(\Lambda^2 - \zeta)^2}{(\Lambda^2 - t)^2}, \tag{2.12}$$

where ζ is the squared mass of the exchanged virtual meson. If the exchanged meson can go on shell, ζ is given by [22]

$$\zeta = t_{max} = m_a^2 + m_1^2 - \frac{1}{2s}\left[(s + m_a^2 - m_b^2)(s + m_1^2 - m_2^2) - \lambda^{1/2}(s, m_a^2, m_b^2)\lambda^{1/2}(s, m_1^2, m_2^2)\right], \tag{2.13}$$

to assure that h is bounded from above by unity.

C. The Thermally Averaged Cross Section

We consider reactions that occur in the central region of relativistic heavy ion collisions. The distributions of the particles composing this hot hadronic matter will be assumed approximately thermal. The cross section, $\sigma(\sqrt{s})$, is a function of a fixed total energy of the initial state. We must sum over $\sigma(\sqrt{s})$ for each value of $\sqrt{s}$ that may occur in the thermal distribution, weighting each cross section's contribution to the sum according to the probability of having that particular configuration. This is denoted as the thermally averaged cross section.

$$\overline{\sigma} = \frac{1}{4m_N^2 m_\phi^2 K_2(\frac{m_N}{T})K_2(\frac{m_\phi}{T})}\int_{\frac{m_N + m_\phi}{T}}^{\infty} dz_1 K_1(z)\lambda(s, m_N^2, m_\phi^2)\sigma_{\phi N}(\sqrt{s}), \tag{2.14}$$

where $s = z^2T^2$, and z is a dimensionless energy variable.

D. The Collision Rate, $\overline{\Gamma}$

The decay rate, Γ, is the probability per unit time that a particle will decay. Using natural units, where $\hbar = c = 1$ one has simply

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[1]Here λ is the standard kinematical function [22] defined by $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$, and sometimes called the triangle function since $\frac{1}{4}\sqrt{-\lambda(a^2, b^2, c^2)}$ gives the area of a triangle with sides a, b, c.

$$\Gamma = \frac{1}{\tau}. \tag{2.15}$$

For the ϕ meson, $\tau_\phi = 15\times10^{-23}$s, $\Gamma_\phi = 4.43$MeV. These are the free space values, indicating the mean lifetime and the decay rate of the ϕ with no external forces acting upon it.

When a particle is situated within the spacetime evolution of a heavy ion collision, for example, these values could change due to the extreme temperatures and densities. There is a significant probability that the ϕ will scatter with other particles in the medium. The scattering randomly kicks the ϕ meson's quantum mechanical phase, effectively giving the distribution a broadened appearance. In equilibrium, however, there are as many ϕ-producing reactions as there are ϕ-reducing ones. Thus, the number does not change, only the distribution does.

The collision rate, $\overline{\Gamma}$, is now approximated by,

$$\overline{\Gamma} = n\,\overline{\sigma} \tag{2.16}$$

where n is the particle density given in Eq. (2.1), and $\overline{\sigma}$ is given by Eq. (2.14). Note that the relative velocity is included within $\overline{\sigma}$. The results for several reactions are included below.

E. The Mean Free Path

Another useful measure of collision rates is the mean free path, λ, which gives a sense of relaxation toward thermalization. It is not the intent of this work to provide a rigorous exploration of the rate equations. Rather, an estimate of the mean free path shall be used as a tool to interpret our findings. We use,

$$\lambda = \tau\,\overline{v} = \frac{\overline{v}}{\Gamma}, \tag{2.17}$$

where $\overline{v}$ is the average velocity of the ϕ in the medium.

The extent in configuration space of the hadronic matter formed in heavy-ion collisions is not precisely known, but a reasonable first estimate comes from a Bjorken model. Upper limits of 10–15 fm are typical. For a given distribution of ϕ's there is finite chance for decay inside, but many will decay outside. We explore consequences of those rare ϕ's that decay inside the medium.

III. RESULTS

The contributions of seven reactions were computed. The cross section for each individual reaction is plotted in Fig. 1. Of the reactions shown, the cross section for the reaction ϕN $\rightarrow$ KΛ with kaon exchange is larger than the others in the region below $\sqrt{s_0} + 1.0$ GeV, where $\sqrt{s_0}$ is the reaction's threshold energy. This energy region is most important since reactions within this domain will be the most common in the nuclear medium. The relatively large cross section for this reaction is due primarily to the large couplings. In the absence of experimental data for the nucleon-kaon-Λ vertex, the coupling is estimated to be the same as the N-Δ-π coupling. The ϕ-K-K coupling is also sizeable. Coupling constants for all reactions are listed in Table I.

The reaction ϕN $\rightarrow \pi$N is the dominant cross section in the region $\sqrt{s} > \sqrt{s_0} + 1.0$GeV. Since reactions at such high $\sqrt{s}$ do not often occur in the medium, a large cross section at extreme $\sqrt{s}$ does not add significantly to the collision rate. The structure emerging in some of the reactions owes to form factors and dissipative details (imaginary pieces) within the various amplitudes. Such pieces are necessary to ensure that the interaction range is never greater than the particle lifetimes allow [23].

The collision rate for each reaction, as given by Eq. (2.16), is depicted in Fig. 2. Again, the largest contribution comes from the ϕN $\rightarrow$ KΛ reaction. The population density of the various initial particles plays a substantial role in determining the collision rate. The density of Δ particles is about one half of the nucleon density at temperature ∼100 MeV, and rises to equality with the nucleon density by about 170 MeV. Thus, while the $\phi\Delta \rightarrow \rho$N cross section is relatively large, its contribution to the collision rate does not rise to prominence until high temperature. Also, the density of the N*(1440) resonance lags behind both the nucleon and Δ densities by an order of magnitude. Thus, the small reaction cross section for ϕN$^* \rightarrow \rho$ N coupled with the small density of N*'s yields a meager collision broadening effect from this reaction.

The total collision broadening from all reactions in Fig. 2 is collected and shown in Fig. 3, along with the previously calculated collision rate due to ϕ + meson reactions [24]. The free space decay rate of the ϕ yields a lifetime of 45 fm/c. Figure 3, however, depicts a broadening of Γ_ϕ due to ϕ baryon interactions by $\simeq$ 10 MeV at a temperature of 170 MeV and a chemical potential of 400 MeV. Recall,

$$\Gamma^{\mathrm{total}} = \Gamma^{\mathrm{decay}} + \Gamma^{\mathrm{collision}} \tag{3.1}$$

where $\Gamma_\phi^{\mathrm{decay}} = 4.43$ MeV. Then the effective width has broadened to ∼14 MeV. Thus, due to baryon collisions alone there is possibly enough evidence to suggest that ϕ becomes kinetically thermal.

The collision broadening due to meson interactions has been computed previously [24]. The estimate for ϕ + meson reactions is plotted along with the current ϕ + baryon prediction in Fig. 3. Through collisions with hadrons the width of the ϕ is broadened by $\simeq$ 24 MeV at 170 MeV with a baryon chemical potential of 400 MeV. One finds a 24 MeV broad ϕ distribution and concludes that there is compelling evidence to suggest that the ϕ meson can fully thermalize even given the brief existence of the reaction heavy ion systems. Employing the estimate of Eq. (2.17), the mean free path of the ϕ is plotted in Fig. 4. At 170 MeV temperature and a chemical potential of 400 MeV, these results indicate that the ϕ has a mean free path of a mere $\simeq$ 5 fm! At the same temperature, and with zero chemical potential, the mean free path rises to $\simeq$ 7 fm. Again, this presents fairly strong evidence for kinetic thermalization of the ϕ.

IV. SUMMARY

We have used effective Lagrangian methods within kinetic theory to describe dynamics of the ϕ meson in a hot and dense, baryon rich nuclear medium. We have shown that the 4.43 MeV free space width of the ϕ is broadened by about 10 MeV through collisions with baryons for conditions typical of high energy heavy ion collisions. These results, coupled

with previous results for meson reactions [24], indicate a broadening of the ϕ width by about 24 MeV in the same conditions. Thermalization for the ϕ meson is a reasonable conclusion.

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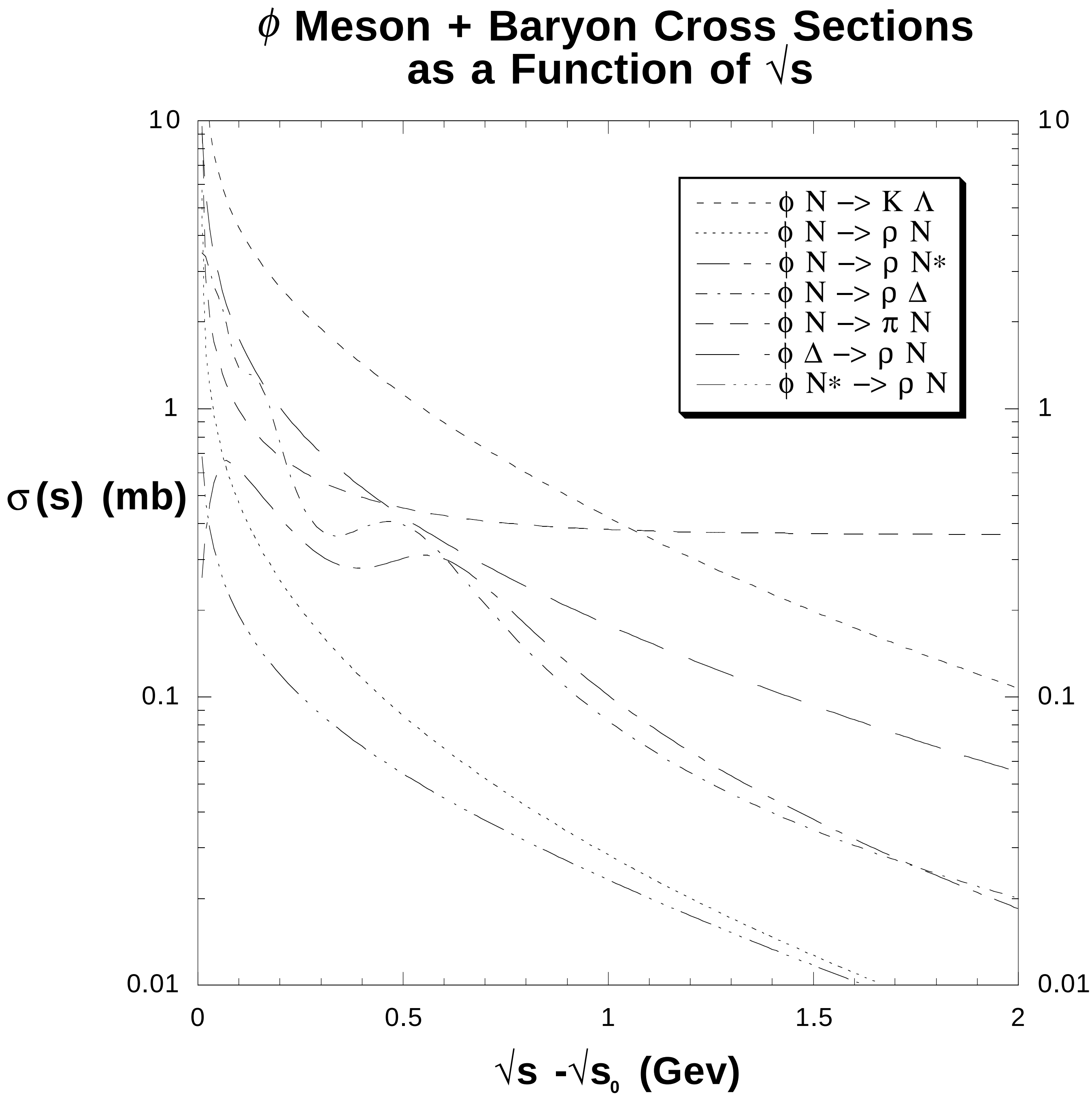

FIG. 1. Cross sections for several ϕ + baryon reactions.

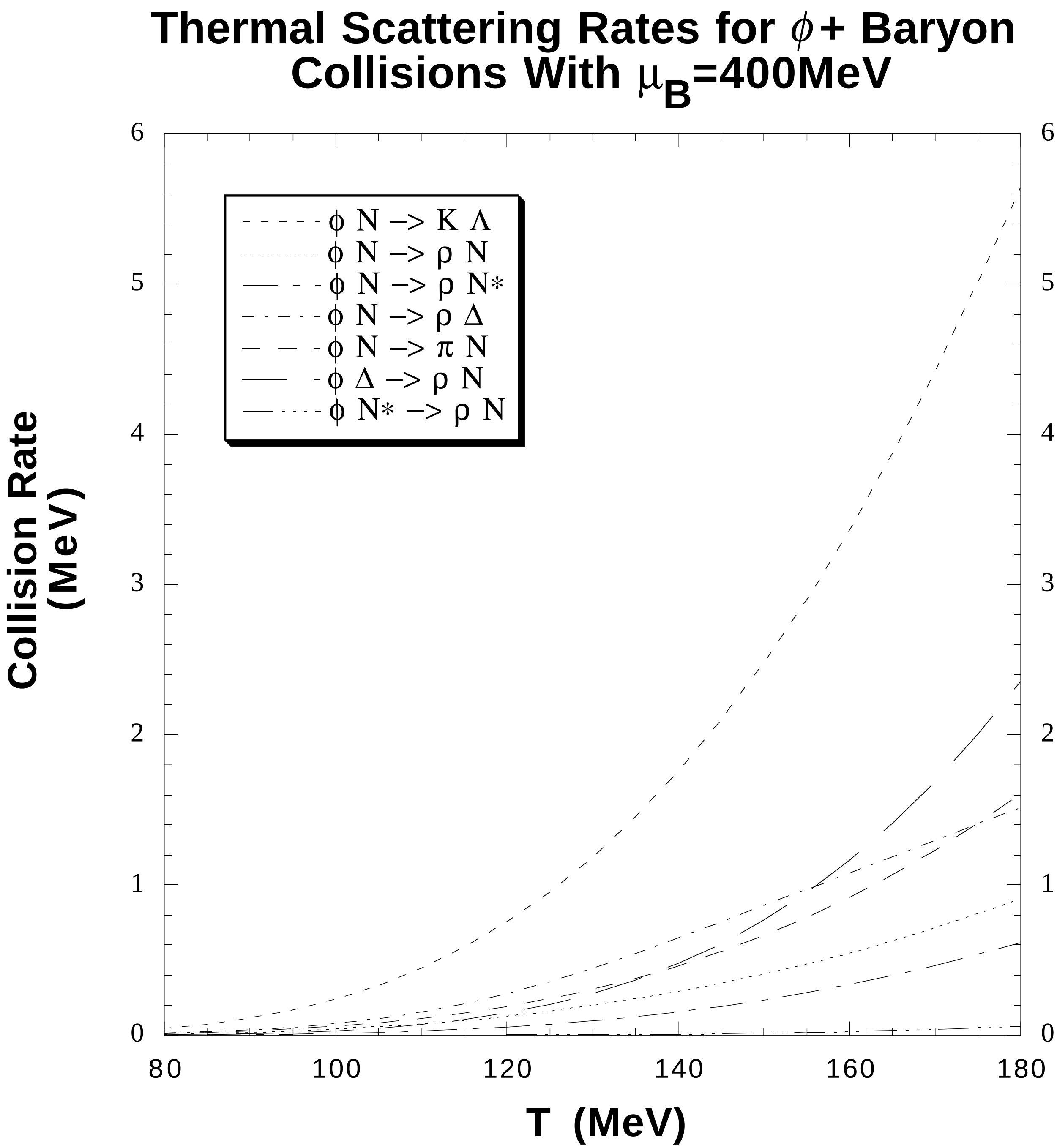

FIG. 2. Partial collision rates of the ϕ meson with baryons in hot, dense matter. This graph assumes a baryon chemical potential of 400MeV.

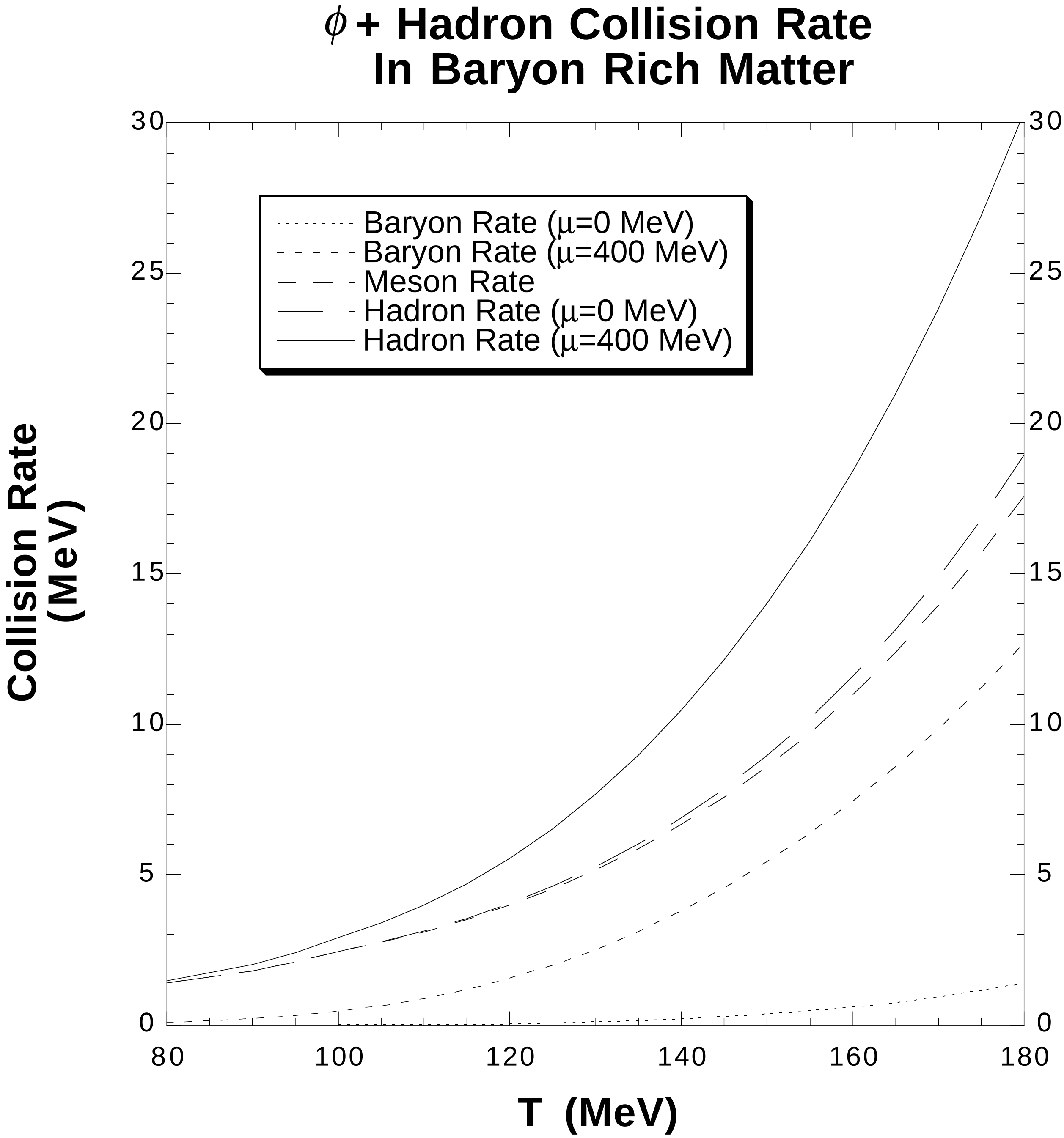

FIG. 3. Collision rate of the ϕ broken down into the previously calculated ϕ + meson reactions, and the current focus, ϕ + baryon reactions.

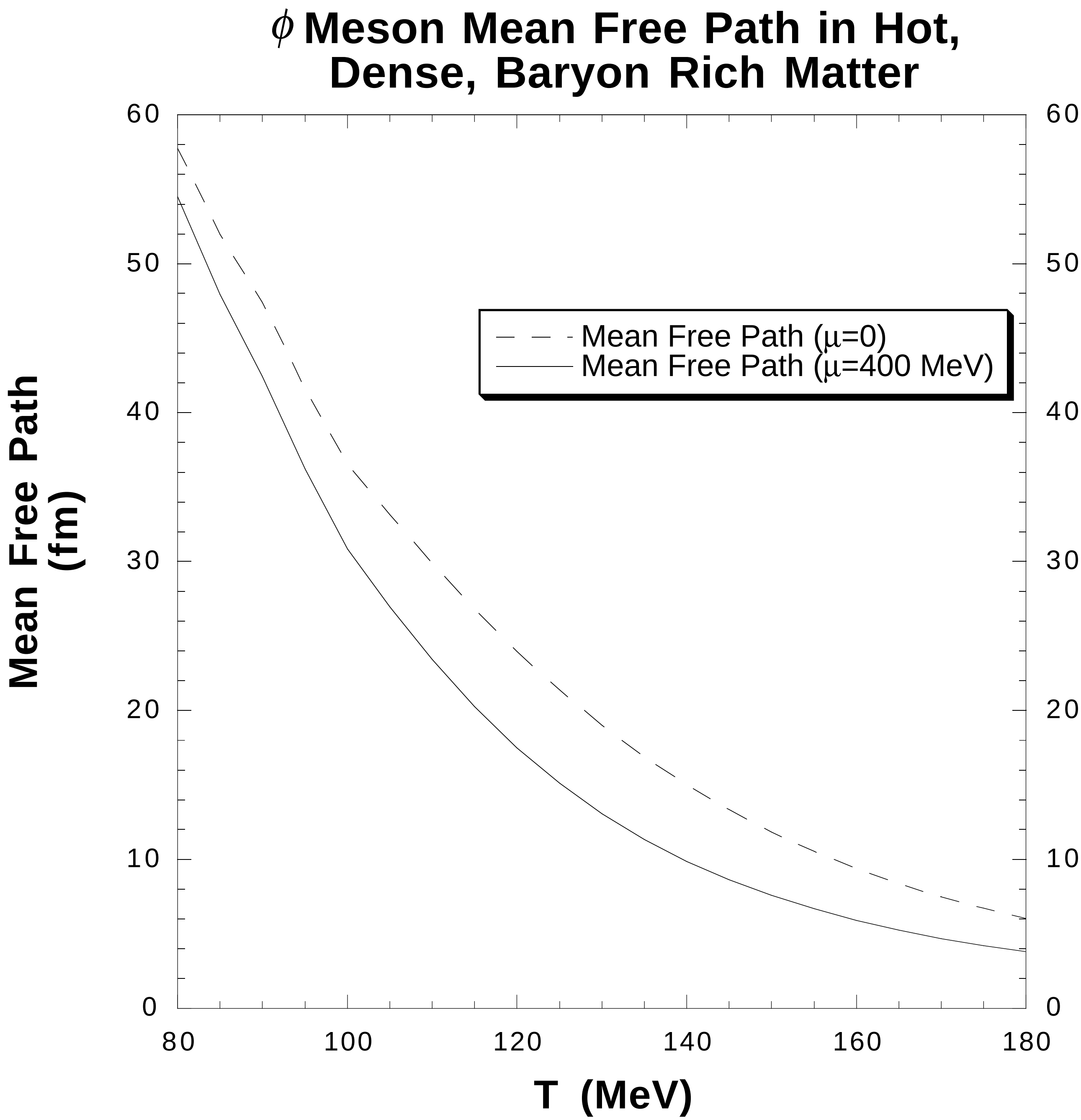

FIG. 4. The mean free path of the ϕ in hot, dense, baryon rich matter. At high temperatures this provides a good indication that the ϕ partially thermalizes.

TABLES

TABLE I. Coupling constants.

Coupling Constant	Numerical Value
$g^2_{N\pi\Delta}/4\pi$	15
$g^2_{\phi\rho\pi}/4\pi$	$5.6\ \times 10^{-3}$
$g^2_{\phi KK}/4\pi$	2.71
$g^2_{NN\rho}/4\pi$ (with $f_{NN\rho}/g_{NN\rho} = 5.0$)	0.85
$g^2_{NN\pi}/4\pi$	14.4
$g^2_{N^*N\pi}/4\pi$	13
$g^2_{NK\Lambda}/4\pi$	15